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REPORT No. 976

MARCH 1956

The Emptying Of A Gun Tube (U)

WILLIAM C. TAYLOR
PETER M. TREUENFELS

DEPARTMENT OF THE ARMY PROJECT No. 5B0302001
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BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 976

WCTaylor/PMTreuenfels/mlu
Aberdeen Proving Ground, Md.
March 1956

THE EMPTYING OF A GUN TUBE (U)

ABSTRACT

An elementary approximate description is proposed for the flow of propellant gases from a gun tube. After a recapitulation of the discussion of the Pidduck-Kent flow, there is presented a simple flow which is conjectured to be, with the unknown correct value of a certain parameter, an accurate asymptotic representation of the (suitably idealized) succeeding flow. This simple flow can be generalized to the emptying of an arbitrary vessel through an orifice and is in turn a generalization of the quasi stationary treatment of the emptying through a small orifice. In the gun it appears to be a good approximation except for a brief interval after the disturbance of the flow by the release of pressure at the muzzle. An example is given in which numerical comparison can be made with experiment and with other calculations.

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The flow of propellant gases from a gun tube becomes important in a number of problems, among which are 1) the estimation of recoil, 2) the design for very rapid fire in which the breech may be opened before the pressure has completely decayed, 3) the design of a muzzle brake or bore evacuator. We are presenting here a simple analytic treatment of the emptying of the tube which may be useful for (1) and (2) but not, in many cases, for such purposes as (3) since it provides a good approximation to the flow except just after the disturbance of the flow by the release of pressure at the muzzle. The aim is to obtain a simple realistic treatment which can be used with confidence in extreme cases and over long intervals of time. Existing methods appear to be sufficiently accurate for most needs in conventional cases. The flow of the gas from the tube is pictured as consisting of two stages. The first stage is governed completely by the initial condition, the flow attained by the expanding column of gas at the instant the base of the projectile emerges from the muzzle. We have supposed this portion of the flow to be the flow described by Love and Pidduck [8] and Kent [5] . In the first section of this report we have recapitulated, perhaps unnecessarily, the discussion of that flow and have, we hope, simplified it somewhat.

The second stage, which we have called the expansion wave for lack of a better term, we have supposed to be governed by the condition which we suppose to exist, that the velocity of efflux of the gases from the muzzle becomes and remains sonic after the exit of the projectile. There are two cases to be considered. If the velocity of the projectile as it emerges is less than the velocity of sound in the gas immediately behind it, we suppose that the velocity at the muzzle increases instantaneously to sonic. If, however, the velocity of the projectile is greater than the velocity of sound in the gas following it, we suppose that within the tube the same Pidduck-Kent flow continues until the velocity of that flow falls to sonic at the muzzle and that thereafter the velocity of the gas at the muzzle remains sonic.

In either case once the velocity at the muzzle is sonic the outflow from the muzzle is more rapid than the formulas describing the Pidduck-Kent flow would indicate. The resulting disturbance of the Pidduck-Kent flow is propagated through that flow at sonic velocity. The equation of the head of this expansion wave (or of any characteristic in the Pidduck-Kent flow) may be expressed as an elementary relation between quantities found by quadrature in the preceding discussion of the Pidduck-Kent flow. The breech pressure is not affected by this expansion wave until the head of this wave reaches the breech. In the meantime, the breech pressure is to be computed from the Pidduck-Kent formulas.

To describe approximately the flow in the expansion wave we have used a method which we judge to be a distinct improvement over earlier treatments. It is a flow in which the Mach number remains constant at each position, the velocity, pressure and density being each a product of a function of position by a function of time. In particular the pressure at any point has the same time dependence as has been found in the well known treatment of the emptying of a large reservoir of gas through a small orifice by assuming quasi-steady flow in the orifice (see, for example, Ackeret [1]). The formulas so obtained have been applied successfully to the emptying of gun tubes (see the historical account in Corner [2] , also Vinti [11] and Lockett [7]) but not happily, since, lacking the partial justification we give here, the effect of the motion of the gas in the tube could not be assessed.

The flow is supposed one-dimensional and homentropic. We suppose the pressure p and density ρ to be connected by the relation (1.3). The effects of friction and of the motion of the tube in recoil are neglected. Accepting the limitations imposed by these assumptions, the shortcomings of the present treatment lie (1) in the possible failure of the Pidduck-Kent flow to provide a sufficiently good approximation to the flow behind the projectile, and (2) probably chiefly in that the flow proposed as a description of the expansion wave does not satisfy the conditions imposed by the conservation relations at the head of the wave. There is available one parameter whose value is to be chosen to fit the expansion wave to the Pidduck-Kent flow, and while we surmise that a proper choice would yield a good asymptotic description of the flow (i.e. after a long time, but before the time the flow in the tube is affected by atmospheric pressure outside) we have not found a truly satisfactory way to determine the value of this parameter. For the purpose of estimating the behavior of the breech pressure we have determined the value of the parameter by imposing the condition that the breech pressure be continuous. A numerical example is included based on experiments performed by Lockett [7]. In these experiments, Lockett measured the variation of breech pressure with time from the instant of shot ejection until about 50 milliseconds afterwards. He then compared these experimental data with values predicted by two methods. In this report we reproduce the results of Lockett, both experimental and theoretical, and adjoin the results of our computations. These results are shown in figures 3 and 4. These are the only pressure records known to us for which careful attention has been paid to the accuracy of the low pressures after shot ejection and which also satisfy consistency requirements. Honesty requires the comment that although our results look very good indeed in these cases, the method cannot be expected to be as good in other cases, in particular for low velocity guns.

Figure 5 shows the predicted behavior of the pressure in the tube in one case. The inconsistency appears there as the discontinuity at the head of the expansion wave. It is clear that the approximation is not good in the early stages of the expansion wave. For problems in which this portion of the flow is important we have employed graphical integration using the characteristics. We hope to present and discuss this elsewhere. Let it suffice to anticipate that discussion only to the extent of remarking that the graphical calculations show, in the cases we have tried, that when the head of the expansion wave has been once reflected at the breech, has traversed the tube again and been swept out the muzzle, the remaining flow in the tube fits the "expansion wave" treatment presented here quite closely.

1. The Pidduck-Kent Flow

The flow discovered by Pidduck [8] and discussed by Kent [5] and Vinti [12, 13] is best described with Lagrangian coordinates. As coordinate to designate any section of the fluid we shall use the mass ψ of gas, per unit area of bore, behind it. Thus $\psi = 0$ is the coordinate of the section of the gas which is (and remains) in contact with the breech. For the gas at the base of the projectile the value of ψ is the total mass C/g of the charge divided by the area A of the bore:

$$\psi = \psi_p = C/Ag.$$

Let the distance of the section ψ from the breech at time t be $x = x(\psi,t)$. The density is then

$$\rho = \rho(\psi,t) = 1\Big/\frac{\partial x}{\partial \psi} \tag{1.1}$$

If the pressure is $p = p(\psi,t)$, the acceleration of the gas is given by

$$\frac{\partial^2 x}{\partial t^2} = -\frac{\partial p}{\partial \psi} \tag{1.2}$$

It is supposed further that the pressure and density are connected by the relation

$$p = k\left(\frac{1}{\rho} - \eta\right)^{-\gamma} \tag{1.3}$$

where the constants η and γ, the covolume and the polytropic exponent, as they are called, of the gas depend only on the composition of the propellant, and may be considered known. The polytropic constant k, on the other hand, specifies the particular adiabatic curve followed during the expansion of the propellant gas after "all burnt".

Combining equations (1.1), (1.2), (1.3), we obtain the differential equation:

$$x_{tt} = -k \frac{\partial}{\partial \psi} \quad (x_{\psi} - \eta)^{-\gamma} \tag{1.4}$$

With the free volume (as it is called)

$$z = x - \eta \psi$$

$$= z \ (\psi, t)$$

as independent variable, equation (1.4) takes the simpler form

$$z_{tt} = -k \ (z_{\psi}{}^{-\gamma})_{\psi} \tag{1.5}$$

To solve this equation by the method of separation of variables, we introduce new independent variables, ϕ and w, and two functions Φ and T by setting

$$\phi = \beta \psi \ , \tag{1.6}$$

$$w = \epsilon t \ , \tag{1.7}$$

$$z = \alpha \quad \Phi(\phi) \ T \ (w), \tag{1.8}$$

where α, β and ϵ are constants still to be determined. We adjust these parameters to simplify the equation resulting from (1.5). Substituting (1.6), (1.7), (1.8) into (1.5) shows that the relation

$$\frac{2\gamma}{\gamma - 1} k \ \alpha^{-\gamma - 1} \ \beta^{-\gamma + 1} \ \epsilon^{-2} = \frac{\gamma - 1}{2} \tag{1.9}$$

leads to

$$\frac{\gamma - 1}{2} \ \Phi^{-1} \ (\Phi')^{-\gamma - 1} \ \Phi'' = \frac{2}{\gamma - 1} T^{\gamma} \ T'' \ ,$$

where primes denote differentiation with respect to ϕ and w, respectively. The common value of both sides of this equation must be constant, and without loss of generality we may assume this constant to be unity. This yields two ordinary differential equations of the second order,

$$(\gamma - 1) \quad (\Phi')^{-\gamma} \, \Phi'' = 2\,\Phi\Phi'$$

and

$$(\gamma - 1)T^{-\gamma} \, T' = 2T' \, T''$$

From the definition, $\Phi(0) = 0$. Designate as $t = 0$ the instant at which the gas is at rest so that $T'(0) = 0$. We set

$$\Phi'(0) = 1 \text{ and } T(0) = 1 \tag{1.10}$$

as two further conditions which serve to specify completely the functions Φ and T. Other non-zero choices lead merely to a different adjustment of numerical factors in Φ and T and their arguments. Integrating, we have then

$$1 - T^{-\gamma+1} = (T')^2 \quad , \tag{1.11}$$

$$1 - (\Phi')^{-\gamma+1} = \Phi^2 \quad , \tag{1.12}$$

and the functions $\Phi(\phi)$ and $T(w)$ are given by

$$\phi = \int_0^{\Phi} (1 - \xi^2)^{\frac{1}{\gamma-1}} \, d\xi \; , \tag{1.13}$$

$$w = \int_1^{T} (1 - \xi^{-\gamma+1})^{-1/2} \, d\xi \tag{1.14}$$

Charts, figures 1 and 2, furnish means for evaluating these functions. For smaller values of Φ and ϕ, expansion of one as a power series in the other is convenient. For much greater precision the tables of Vinti and his collaborators [12, 13] may be used.

The parameters k, β, α, and ϵ must be evaluated to suit the individual case. In the type of problem under consideration the muzzle velocity V_m and the weight W of the projectile are known, as are the weight C and chemical energy of the propelling gas and its thermodynamic properties, γ and $\bar{\eta}$. The values of the parameters can be obtained from these as is shown below. It must be remembered however that the solution has not been fitted to whatever initial conditions existed in the gas when the solid propellant was all burned. The adequacy of this treatment depends on the conjecture that the asymptotic behavior of the solution is independent of the initial conditions (and, of course, from another point of view, it depends on the experimental verification).

Let the subscript p refer to the base of the projectile. From (1.6) we have

$$\beta = \frac{\phi_p}{\Psi_p} = gA\ \phi_p/C \tag{1.15}$$

where A is the area of the cross section of the bore.

The force on the base of the projectile determines its acceleration in the fashion:

$$Ap = \frac{W'}{g}\ x_{tt} = -\frac{W'}{g}\ p_\psi ,$$

where (1.2) has been used and W' is the weight of the projectile, adjusted to account for friction. Inserting (1.12) and (1.15) there results after some reduction,

$$\phi_p \Phi_p \left(1 - \Phi_p^2\right)^{-\frac{\gamma}{\gamma - 1}} = \frac{\gamma - 1}{2\gamma}\ \frac{C}{W'} , \tag{1.16}$$

which may be solved simultaneously with (1.13) for ϕ_p and Φ_p as functions of $\frac{C}{W'}$. Then β may be found from (1.15).

Let the subscript e refer to the instan the base of the projectile emerges from the tube. The quantities T_e', T_e given in (1.18) below are found by counting up the energy. The total energy released by the propellant is $\frac{FC}{(\gamma - 1)}$, where F is the specific force of the propellant. The kinetic energy of the projectile is $\frac{1}{2g} W'V^2$ and the ratio of these is the efficiency,

$$\theta = \frac{\gamma - 1}{2}\ \frac{W'}{gFC}\ V^2 , \tag{1.17}$$

and θ_e may be computed. The internal energy of the gas is

$$\frac{A}{\gamma-1}\int_0^{z_p} p\,dz = \frac{Ak}{\gamma-1}\int_0^{z_p} z_\psi^{-\gamma}\quad dz = \frac{\gamma-1}{4}\frac{A\alpha^2\epsilon^2}{\beta\gamma}T^{-\gamma+1}\int_0^{\Phi_p}(1-\Phi^2)^{\frac{\gamma}{\gamma-1}}\,d\Phi.$$

The kinetic energy of the gas is

$$\frac{A}{2}\int_0^{\psi_p} x_t^2\ d\psi = \frac{A}{2\beta}\quad \alpha^2\epsilon^2 T'^2\int_0^{\Phi_p}\Phi^2(1-\Phi^2)^{\frac{1}{\gamma-1}}\quad d\Phi$$

We may write

$$(\text{Total energy}) = \begin{pmatrix}\text{Kinetic energy of}\\ \text{the projectile}\end{pmatrix} + \begin{pmatrix}\text{Kinetic energy}\\ \text{of the gas}\end{pmatrix} + \begin{pmatrix}\text{Internal energy}\\ \text{of the gas}\end{pmatrix}$$

If the preceding expressions are inserted and some reduction carried out there results

$$T'^2_e = \frac{\gamma-1}{3\gamma-1}\quad \theta_e\left(1+\frac{C}{W'\Phi_p^2}\right) \tag{1.18}$$

whence we find T_e by means of (1.11). The total volume of the gun is, from (1.8)

$$\eta\,\frac{C}{g} + \alpha\Phi_p\,T_e\,A\ , \tag{1.19}$$

This furnishes the value of α. Equation (1.8) shows also that

$$V_m = \alpha\,\epsilon\,\Phi_p T'_e \tag{1.20}$$

which yields ϵ. Finally, (1.9) gives the value of k.

For convenience of reference, we collect here expressions for x_t, ρ and c as functions of the Eulerian coordinates x and t.

We first calculate Φ and T as functions of x and t. To this end we first find w by (1.7) and T by (1.14). Then we calculate ϕ and Φ by using (1.8) and (1.13) simultaneously. Equation (1.8) gives us

$$x_t = \alpha\,\epsilon\quad T', \tag{1.21}$$

while (1.1) and (1.8) yield

$$\rho = \left[\alpha\, \beta \Phi' T + \eta\right]^{-1} \tag{1.22}$$

The pressure p is, by (1.3),

$$p = k\, (\alpha\, \beta \Phi' T)^{-\gamma} \tag{1.23}$$

In particular, at the breech $\Phi' = 1$ and

$$p_{breech} = k\, (\alpha\, \beta\, T)^{-\gamma} \tag{1.24}$$

Finally, the sonic velocity c is found by means of the relation

$$c^2 = \frac{dp}{d\rho}$$

to be

$$c = (\gamma k)^{1/2}\, (\alpha\, \beta \Phi' T)^{-\frac{\gamma - 1}{2}} \left[1 + \frac{\eta}{\alpha\, \beta \Phi' T}\right]. \tag{1.25}$$

If the muzzle velocity is less than sonic velocity, an expansion wave starts to travel from the muzzle toward the breech at the instant of shot ejection. However, if the muzzle velocity is greater than the sonic velocity, there will be a time interval during which the propellant gas velocity at the muzzle drops to sonic. We shall be interested in calculating the duration of this interval. Let Φ_s and T_s denote the values of Φ and T respectively at the muzzle and at the instant when $x_t = c$. From (1.21) and (1.25) we obtain one of the conditions to be satisfied by Φ_s and T_s:

$$\Phi_s\, T_s' = \frac{\gamma - 1}{2}\, (\Phi_s' T_s)^{-\frac{\gamma - 1}{2}} \left[1 + \frac{\eta}{\alpha\, \beta \Phi_s' T_s}\right]. \tag{1.26}$$

The second condition to be satisfied by Φ_s and T_s is based on the fact that the expansion wave starts at the muzzle. Hence by (1.8):

$$x_m = \frac{1}{\beta}\, \phi_s + \alpha \Phi_s\, T_s \quad , \tag{1.27}$$

where x_m is the equivalent muzzle distance. Solving these two equations simultaneously gives us the desired values of Φ_s and T_s.

The head of the expansion wave, the initial disturbance of the Pidduck flow, is propagated at sound speed with respect to the gas, that is, along a characteristic of that flow. See, for example, Courant-Friedricks [3] , p. 106. Its progress is therefore governed by the equation

$$\frac{d\psi}{dt} = - \rho c \; .$$

If this relation is expressed in terms of the variables Φ and T' it can readily be integrated. The result is

$$\sin^{-1} \Phi = -\sin^{-1} T' + \text{constant}. \qquad (1.28)$$

The constant may be evaluated by substituting the values Φ_s and T'_s . In particular, at the time at which the expansion wave arrives at the breech

$$T' = T'_b = \sin\,(\sin^{-1} T'_s + \sin^{-1} \Phi_s)\,. \qquad (1.29)$$

This should be an improvement on a method used by Kent [6] to estimate the time of the arrival of the expansion wave at the breech.

2. The Expansion Wave

The differential equations (2.3) and (2.4) below, for nearly one-dimensional compressible flow in a tube of variable cross section, have solutions in which the velocity and the speed of sound are, respectively,

$$u = \frac{X(x)}{t - t_o} \quad , \qquad (2.1)$$

and

$$c = \frac{L(x)}{t - t_o} \qquad (2.2)$$

where $X(x)$ and $L(x)$ denote functions depending only upon the distance x along the gun tube, t_0 being a suitably chosen constant. A solution in which u vanishes at the breech and u = c at the muzzle is used to describe the later stage of the emptying of a gun tube. In the case of constant cross-sectional area it is found to be possible to express the functions X and L parametrically in terms of quadratures.

A similar treatment may be applied to the emptying of a vessel of any shape through an orifice small enough that we can accept an arbitrary specification of the surface over which the velocity is to be sonic. We have not investigated the possibility of proving the conjecture that the flow from a tube into a vacuum has asymptotically the character described here. It would appear to be somewhat easier to investigate than the corresponding conjecture about the asymptotic character of the flow behind a projectile. For the emptying of a vessel of a shape other than a tube the problem would include inferring the shape of the surface over which the velocity is sonic.

The differential equations which govern the almost one-dimensional flow of a non-viscous gas along a tube of slightly and slowly varying cross-section A = A(x) are, in Eulerian form,

$$A\,\rho_t + (\rho A u)_x = 0\ , \tag{2.3}$$

$$u_t + u\,u_x = \frac{-1}{\rho}\,p_x\ , \tag{2.4}$$

where u = u (x,t) is the velocity of the gas and the other letters represent the same physical quantities as before. It will be supposed that the pressure and density are connected by the relation

$$p = k\,\rho^{\gamma} \tag{2.5}$$

It is convenient to introduce the sonic velocity c into equations (2.3) and (2.4) instead of the density. We have

$$c^2 = dp/d\rho = \gamma k \rho^{\gamma-1} = \gamma k^{1/\gamma}\ p^{\frac{\gamma-1}{\gamma}} \tag{2.6}$$

This yields

$$c_t + u\,c_x = -\frac{\gamma - 1}{2}\;cu_x - \frac{\gamma - 1}{2}\;cu\,\frac{A'}{A} \tag{2.7}$$

$$u_t + u\,u_x = -\frac{2}{\gamma - 1}\,c\,c_x \tag{2.8}$$

Introducing (2.1) and (2.2) there results

$$s(L^2 - X^2)\,X' = L^2 - sX^2 - sL^2X\,\frac{A'}{A}$$

$$(L^2 - X^2)\,L' = -(1-s)\,LX + sLX^2\,\frac{A'}{A} \tag{2.9}$$

where

$$s = \frac{\gamma - 1}{2}$$

For a given $A(x)$ the system (2.9) could be integrated numerically. Problems such as we are here concerned with would have two point boundary conditions which might cause a bit of trouble. We have considered only the case in which A is constant. A similar treatment could be given for the case in which LA'/A is constant. If this constant has the value $\frac{1 - s}{s}$, the right hand sides of (2.9) vanish at $X = L$ and no singularity appears.

When A' is identically zero an integral of (2.9) can be found and the solution can be expressed parametrically in terms of a quadrature. It is convenient to introduce as parameter the Mach number,

$$M = M(x) = X/L. \tag{2.10}$$

Equations (2.9) may then be written

$$X' = \frac{1 - s\,M^2}{s(1 - M^2)}\;,$$

$$L' = \frac{-(1 - s)M}{1 - M^2}\;. \tag{2.11}$$

Also

$$LM' = X' - ML'$$

$$= \frac{1 - s^2M^2}{s(1 - M^2)} \tag{2.12}$$

and

$$\frac{L'}{LM'} = \frac{1}{L}\frac{dL}{dM} = \frac{-s(1-s)\,M}{1 - s^2M^2}$$

Integrating,

$$L = L_o\,(1 - s^2M^2)^{\frac{1-s}{2s}} \tag{2.13}$$

where L_o is constant. Then

$$X = ML = L_oM\,(1 - s^2M^2)^{\frac{1-s}{2s}} \tag{2.14}$$

Inserting (2.13) in (2.12) and integrating, there results, since $M = o$ at $x = o$,

$$x = sL_o \int_0^M (1 - M^2)\,(1 - s^2M^2)^{\frac{1-3s}{2s}}\,dM \tag{2.15}$$

The value of L_o is determined by requiring that $M = 1$ at the muzzle, $x = b$, say.

The integral (2.15) can be expressed in terms of elementary functions when $\frac{1}{s}$ is an integer, $\gamma = 5/3,\ 6/4,\ 7/5$, etc. With a little manipulation it can be evaluated also from Chart I. Since s^2 is small, a satisfactory evaluation is obtained by expanding the second factor of the integrand in powers of M^2. Errors of only a fraction of one percent are made by omitting the second factor. Then

$$x = sL_o\,\left(M - \frac{M^3}{3}\right) \tag{2.16}$$

and for $M = 1$,

$$x = b = \frac{2}{3} s L_o \tag{2.17}$$

Expanding (2.13) similarly we have, approximately,

$$L = L_o \left(1 - \frac{s(1-s)}{2} M^2\right) \tag{2.18}$$

and

$$X = M L_o \left(1 - \frac{s(1-s)}{2} M^2\right) \tag{2.19}$$

In this approximation the character of the singularity at the muzzle is unchanged. Consideration of the first neglected terms indicates that the maximum error in the pressure introduced in our examples by the use of (2.15) instead of (2.16) would be at most about 3%, certainly negligible in the presence of other crudities of the treatment. Actually the plotted values were computed by a different approximation subject to errors of the same size.

3. Sample Calculations

Lockett [7] has measured pressures at the breech of the 3"/70 after the exit of the projectile and to the time when the pressure has dropped to perhaps 50 psi. He has also computed estimates of the pressure by the schemes which have been proposed by Rateau.

We shall now carry out computations of the breech pressures and of pressure profiles by the method proposed above and compare the results with the pressures measured and computed by Lockett. The results of these computations are shown in figures 3 and 4, together with the pressures measured and computed by Lockett.

We shall now explain the details of our computations for rounds No. 143-151. We begin by listing the data given by Lockett, converting them to the inch-pound (force)-second system. The charge is described by

$$\gamma = \text{ratio of specific heats} = 1.3,$$

η = covolume = 1.06 cm^3/g (in volume/weight units)

= 11,300 $in^4\ lb^{-1}\ sec^{-2}$ (in volume/mass units),

F = force of the powder = 52.9 long tons/sq. in. per g/cm

= $3.28 \cdot 10^6$ inches,

C = weight of charge = 10 lbs, 2 oz, 8 drams

= 10.2 lbs.

The gun is described by:

A = cross-sectional area of the bore, = 7.07 sq. in.

W = weight of the projectile, = 15 lbs.

V_m = muzzle velocity, = 3370 ft/sec = 40,400 in./sec (this is an average of the five measurements obtained for rounds 143, 145, 147, 149 and 151).

U_t = total volume of the gun, consisting of the volume of the gun barrel and of the volume of the chamber, = 1718 cu. in.

Note: $U_t = U_{ch} + A\ X_m$, where U_{ch} is the chamber volume, A the bore area and X_m the travel of the projectile.

We shall now compute the constants occurring in the theory of the Pidduck-Kent flow, i.e. α, β, and ϵ. These constants will enable us to compute the pressure, density, propellant gas velocity and sonic velocity in that part of the flow which is presumed to be governed by the Pidduck-Kent theory.

We begin by calculating the values of Φ and ϕ at the base of the projectile. To this end we solve (1.13) and (1.16) simultaneously. The symbol W' of (1.16) designates the projectile weight after it has been adjusted to account for energy losses due to inertial and dissipative resistance. We have found it practical to multiply W by 1.05 to obtain W'. This yields W' = 15.8 lbs. In order to solve (1.13) and (1.16) simultaneously, we may proceed as follows: we first guess at a value for Φ_p, say $\Phi_p^{(1)} = \left[\frac{\gamma - 1}{2\gamma}\frac{C}{W'}\right]^{1/2}$. We use chart 1 to find the value of ϕ corresponding to $\Phi_p^{(1)}$, say $\phi_p^{(1)}$. We then calculate the left-hand side of (1.16), using $\phi = \phi_p^{(1)}$ and $\Phi = \Phi_p^{(1)}$. We then guess a second value for Φ_p and repeat this process. By linear interpolation we then find a third guess for Φ_p, and we iterate in this manner as often as may be necessary. This yields $\Phi_p = 0.246$ and $\phi_p = 0.230$.

We next find the constant β by means of equation (1.15)

$$\beta = 61.9 \text{ in}^3 \text{ lb}^{-1} \text{ sec}^{-2} .$$

To compute the constants α and ϵ, we begin by finding the thermodynamic efficiency θ at the instant of shot ejection, by means of (1.17): $\theta_e = 0.301$. Next, we find T' at the instant of shot ejection by means of (1.18): $T'_e = 0.602$. It follows from (1.11) that $T_e = 4.47$.

We are now ready to calculate the constants α, ϵ and k. Equation (1.19) shows that $\alpha = 183$ inches, and equation (1.20) yields $\epsilon = 1500 \text{ sec}^{-1}$. Finally equation (1.9) furnishes $k = 2.12 \cdot 10^{10} \text{ in}^{4\gamma - 2} \text{ lb}^{-\gamma + 1} \text{ sec}^{-2\gamma}$.

These constants enable us to calculate the propellant gas velocity x_t, the density ρ, the pressure p and the sonic velocity c as functions of the distance x and the time t during the Pidduck-Kent flow [cf. (1.21)-(1.25)].

However, the Pidduck-Kent flow ultimately ceases when the rarefaction wave, starting at the muzzle, has reached the breech. Hence, in order to determine the time-space interval during which the Pidduck-Kent flow obtains, we shall now investigate the progress of the expansion wave.

We begin by calculating the time when the expansion wave starts at the muzzle. If the muzzle velocity is sub-sonic, the expansion wave will start at the muzzle at the instant of shot ejection. However, if the muzzle velocity is super-sonic, the expansion wave will not start upstream until the velocity at the muzzle of the escaping powder gases has dropped to sonic. To decide which case we are dealing with, we shall calculate the sonic velocity at the muzzle at the instant of shot ejection. For this purpose we use (1.25), which yields c_e = 37,500 in./sec. Comparing this with the given muzzle velocity V_m = 40,400 in./sec, we see that the muzzle velocity in the present numerical case under consideration is supersonic.

To find T_s, which is the value of T characterizing the instant when the expansion wave starts at the muzzle, we solve (1.26) and (1.27) simultaneously. To this end we guess at a value for Φ_s, say $\Phi_s^{(1)}$, and use chart 1 to find the corresponding value of ϕ, say $\phi_s^{(1)}$. Using (1.27), we find the corresponding value of T, say $T_s^{(1)}$. This enables us to find the right-hand side of (1.26). A new estimate for Φ, say $\Phi_s^{(2)}$, is now obtained by dividing this right-hand side of (1.26) by $T_s'^{(1)}$. This process is repeated as often as necessary. We obtain $\Phi_s = 0.210$, $T_s = 5.37$ and $T_s' = 0.629$.

Having started at the muzzle, the expansion wave now travels towards the breech. The formulae relating to the Pidduck-Kent flow still remain applicable to that portion of the propellant gas which is included between the head of the expansion wave and the breech. However, this applicability will cease, naturally, when the expansion wave reaches the breech. We therefore calculate next T_b, which is the value of T characterizing the instant when the expansion wave arrives at the breech. To this end we use (1.29), which can be put into the following more convenient form, which is recommended when trigonometric tables are not available:

$$T'_b = \Phi_s \left(1 - T'^2_s\right)^{1/2} + T'_s \left(1 - \Phi_s^2\right)^{1/2}$$

This furnishes $T'_b = 0.779$. Hence, by (1.11), $T_b = 22.4$.

We have thus characterized, by means of the corresponding values of T, three instants: shot ejection, start of the expansion wave at the muzzle, and arrival of the expansion wave at the breech, viz., we have found $T_e = 4.47$, $T_s = 5.37$ and $T_b = 22.4$.

To find the values of the time t at these three instants, we use chart 2. Let us consider the instant of shot ejection. Here $T'^2_e = 0.362$. Using chart 2, we find $w(T_e) = 8.64$. Finally we use equation (1.7) to calculate $t_e = 5.78$ msec. Similarly, we obtain $t_s = 6.76$ msec, $t_b = 22.4$ msec, all referred to the instant $t = 0$ corresponding to $T = 1$. For practical purposes, however, it is more convenient to refer events to the instant of shot ejection. We therefore form the differences, to find $t_s - t_e = 0.985$ msec, $t_b - t_e = 16.6$ msec. Thus the expansion wave starts 0.985 msec after shot ejection, and arrives at the breech 16.6 msec after shot ejection.

We are now ready to discuss the computation of pressure profiles whenever the formulae for the Pidduck-Kent flow are applicable.

In the present case it was decided to calculate the pressure at 4 msec intervals from 0 to 16 msec after shot ejection; and at 60 inch intervals along the gun tube from 0 to 180 inches (measured from the breech), as well as at the muzzle, which corresponds to $U_t/A = 243$ inches from the breech.

We begin by calculating the progress of the expansion wave along the gun barrel from the muzzle to the breech. To this end we first calculate T corresponding to each instant of time t for which a pressure profile is desired. Let us carry out a sample calculation for the instant of 8 msec after shot ejection. This instant is suitable, since $t_s - t_e < 8 \cdot 10^{-3} < t_b - t_e$. Denoting this instant by t_8, we have $t_8 = t_e + 8 = 13.8$ msec, and hence by (1.7) and (1.14), $T_8 = 12.6$. We rewrite (1.28) in the form

$$\Phi = T_b' \left(1 - T'^2\right)^{1/2} - T' \left(1 - T_b'^2\right)^{1/2}$$

Then $\Phi_8 = .0748$. This is a measure of the mass of powder gas (per unit area of the bore) included between the breech and the head of the expansion wave at the instant t_8. The distance x from the breech to the head of the expansion wave at this instant can now be calculated by equation (1.8) yielding $\hat{x}(t_8) = 186$ inches. We compute $\hat{x}(t)$ in this manner for the prescribed values of t satisfying $t_s \leq t \leq t_b$.

We are now ready to compute the pressure profiles for that portion of the flow which is described by the Pidduck-Kent formula (1.23). For values of t satisfying $t_e \leq t \leq t_s$, we can find the pressure p as a function of the distance x (measured from the breech) for all values of x along the gun tube, i.e. for all x satisfying $0 \leq x \leq U_t/A$. For all values of t satisfying $t_s \leq t \leq t_b$, we can find p(x) for all x satisfying $0 \leq x \leq \hat{x}(t)$. But the flow in the remainder of the semi-infinite strip of the x - t plane $0 \leq x \leq U_t/A$, $t_e \leq t$ is not described by the Pidduck-Kent formulae.

Let (x,t) denote a point of this semi-infinite strip for which the Pidduck-Kent formulae are applicable. To calculate the pressure p(x,t), we begin by finding Φ corresponding to the given point (x,t). As an example, let us consider the point 120 inches from the breech a and 8 msec after shot ejection. Since $120 < 186 = \hat{x}(t_8)$, the Pidduck-Kent formulae are applicable at this point. To find Φ corresponding to (x_{120}, t_8), we use (1.8), whence $\Phi(x_{120}, t_8) = .0483$. The pressure is now found by equation (1.23): $p(x_{120}, t_8) = 4200$ lbs/sq. in. The pressure at the instant t_8 is thus computed for all desired values of x, including e.g. the pressure at the head of the expansion wave, by setting $x = \hat{x}(t_8)$. In our example we obtain p = 4140 psi for this pressure.

Pressure profiles can thus be calculated for all desired values of t, for the Pidduck-Kent portion of our flow.

In particular, we need to know the pressure at the breech at the instant the expansion wave arrives there, i.e. at t_b. Since in our example we have $T_b = 22.4$, (1.24) shows that the breech pressure at t_b is $p(0, t_b) = 2010$ psi.

The formulae for the flow of the second type are assumed to apply to that portion of the flow which is not described by the formulae of the Pidduck-Kent flow, i.e. for $t_s \leq t \leq t_b$, $\hat{x}(t) \leq x \leq U_t/A$, and for $t_b \leq t$, $0 \leq x \leq U_t/A$.

Various conditions could be imposed in order to effect the determination of t_o. Since we are interested in variations of the breech pressure with time, we have imposed the requirement that the breech pressure be continuous at the instant of arrival of the expansion wave at the breech, $t = t_b = .0224$ sec. We had found above that $p(0, t_b) =$ 2010 psi and the corresponding velocity of sound is $c_b = 25{,}200$ in./sec. Since we are neglecting covolume excess from this point on, the manner in which c should be computed is indeterminate to the extent of about 2%. Fortunately the effects of this, ultimately 17%, become large only at later times when the percentage accuracy of our determination and of the measurements is equally reduced. From (2.17) we find

$$L_o = \frac{3b}{2s} = \frac{3U_t}{.3A} = 2430 \text{ in.}$$

and (2.2) with x = M = 0 gives

$$c = c_b = \frac{L_o}{t_b - t_o}$$

from which $t_o = -.0741$ sec.

Having determined t_o, we are ready to calculate the desired pressure profiles. Let us calculate e.g. the pressure at t_{20}, which is the instant 20 msec after shot ejection, and at $x_{90} = 90$ inches from breech. We note first that $t_e = 5.78$ msec implies $t_{20} = 25.8$ msec. From (2.16)

$$M - \frac{M^3}{3} = \frac{x}{s\,L_o} = .247$$

and we may solve for $M = .252$. The pressure is, using (2.2) and (2.13)

$$p = p(0,t_b)\left(\frac{c}{c_b}\right)^{\frac{2\gamma}{\gamma-1}}$$

$$= p(0,t_b)\left(\frac{t_b - t_o}{t - t_o}\right)^{\frac{2\gamma}{\gamma-1}}\left(1 - s^2 M^2\right)^{\frac{\gamma(3-\gamma)}{(\gamma-1)^2}} \qquad (3.1)$$

$$= 1450 \text{ psi},$$

when the values above are inserted.

As a second example, let us compute the pressure at the head of the expansion wave 8 msec after shot ejection. By our previous calculations we had found that $\hat{x}(t_8) = 186$ inches. Hence by equations (2.16) and (3.1) $p\left[\hat{x}(t_8),\ t_8\right] = 3770$ psi, as compared to 4140 psi calculated at the same instant and at the same point by the Pidduck-Kent formulae.

Pressure profiles can thus be calculated for all values of $t \geq t_e$, either by the Pidduck-Kent formulae or by the formulae for the flow of the second type. The former hold for $t_e \leq t \leq t_s$ and all x, and for $t_s \leq t \leq t_b$ and $0 \leq x \leq \hat{x}(t)$; while the latter are to be used for $t_s \leq t \leq t_b$ and $\hat{x}(t) \leq x \leq U_t/A$, and for $t \geq t_b$ and all x.

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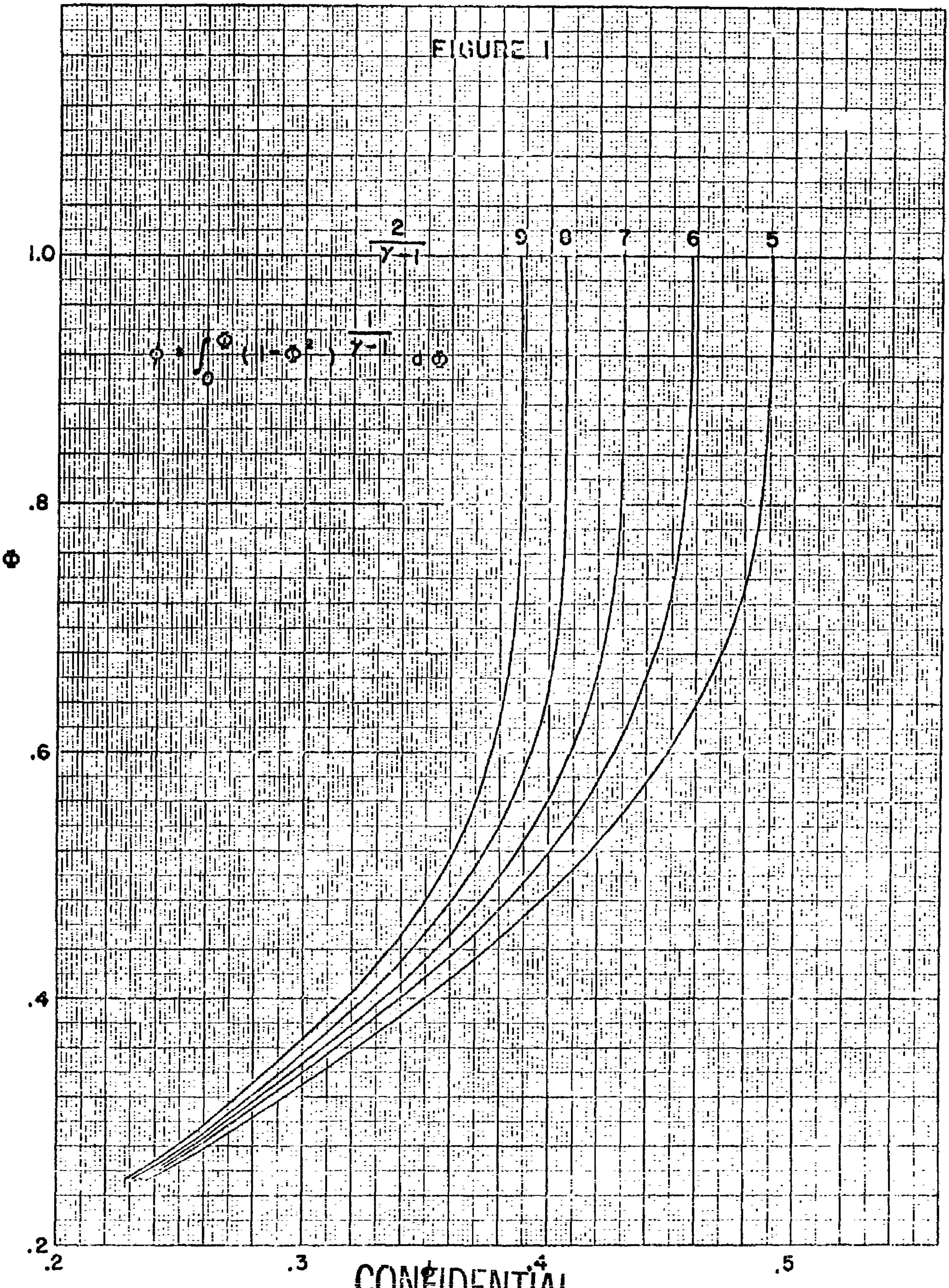
FIGURE 1
$\frac{2}{\gamma+1}$
.9
.8
.7
.6
.5
$\varphi = \int_0^{\Phi} (1-\Phi^2)^{\frac{1}{\gamma-1}} d\Phi$
1.0
.8
.6
.4
.2
Φ
.2
.3
.4
.5

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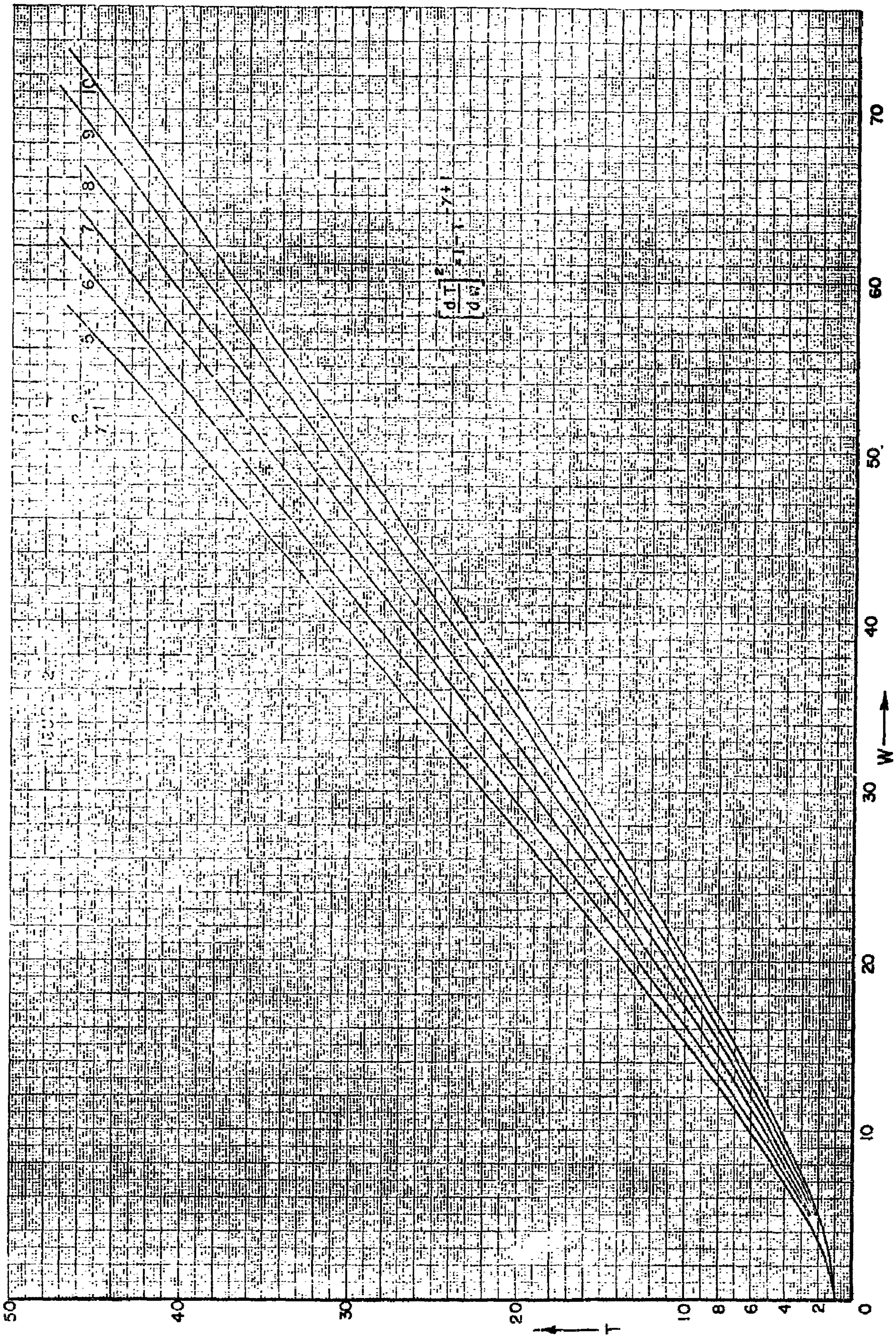

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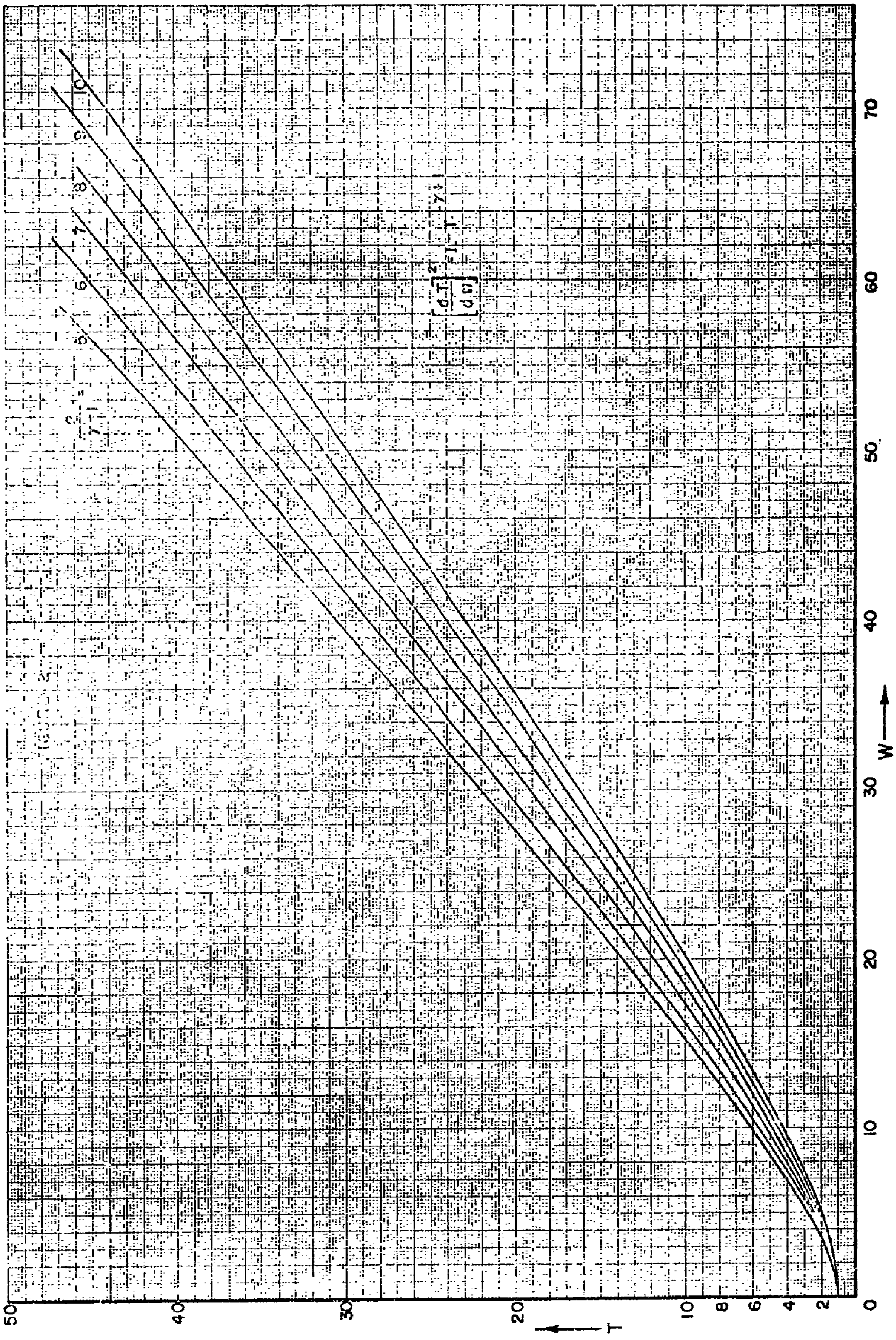

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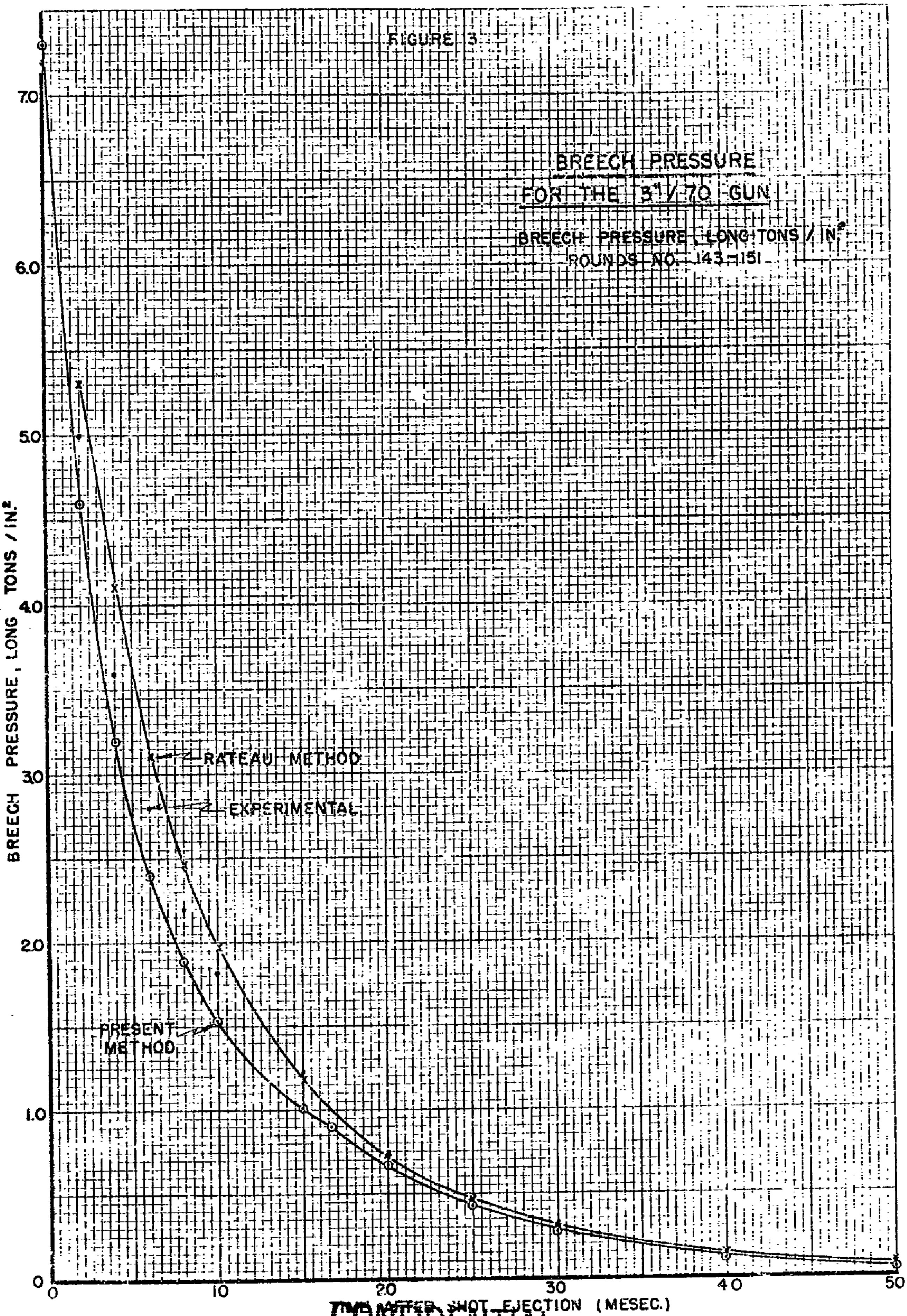
FIGURE 3
BREECH PRESSURE
FOR THE 3"/70 GUN
BREECH PRESSURE, LONG TONS / IN.²
ROUNDS NO. 143-151
RATEAU METHOD
EXPERIMENTAL
PRESENT METHOD
BREECH PRESSURE, LONG TONS / IN.²
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
0
10
20
30
40
50
TIME AFTER SHOT EJECTION (MESEC.)

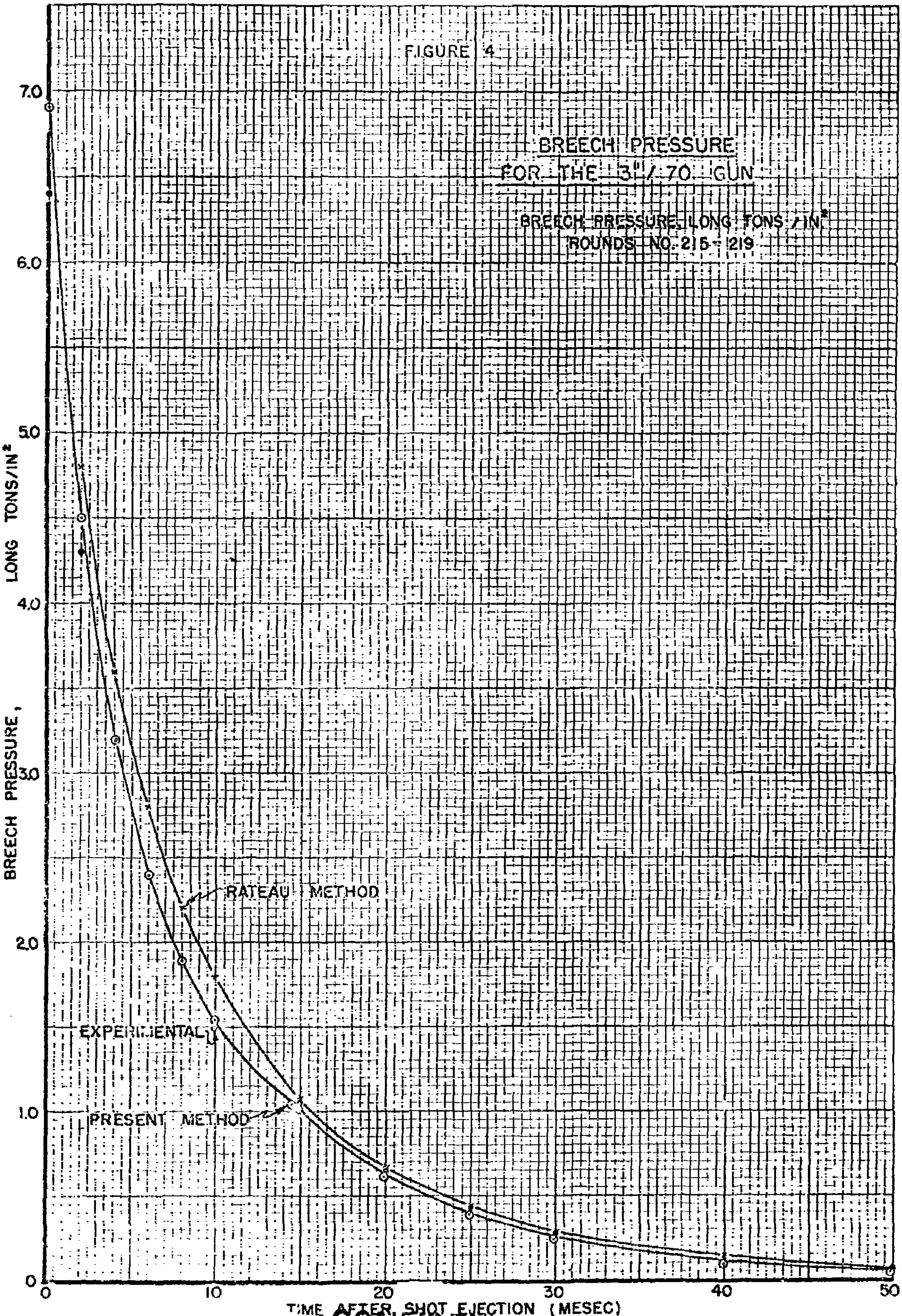
FIGURE 4
BREECH PRESSURE
FOR THE 3"/70 GUN
BREECH PRESSURE, LONG TONS / IN²
ROUNDS NO. 215 - 219
RATEAU METHOD
EXPERIMENTAL
PRESENT METHOD
BREECH PRESSURE, LONG TONS/IN²
TIME AFTER SHOT EJECTION (MESEC)
0
10
20
30
40
50
1.0
2.0
3.0
4.0
5.0
6.0
7.0

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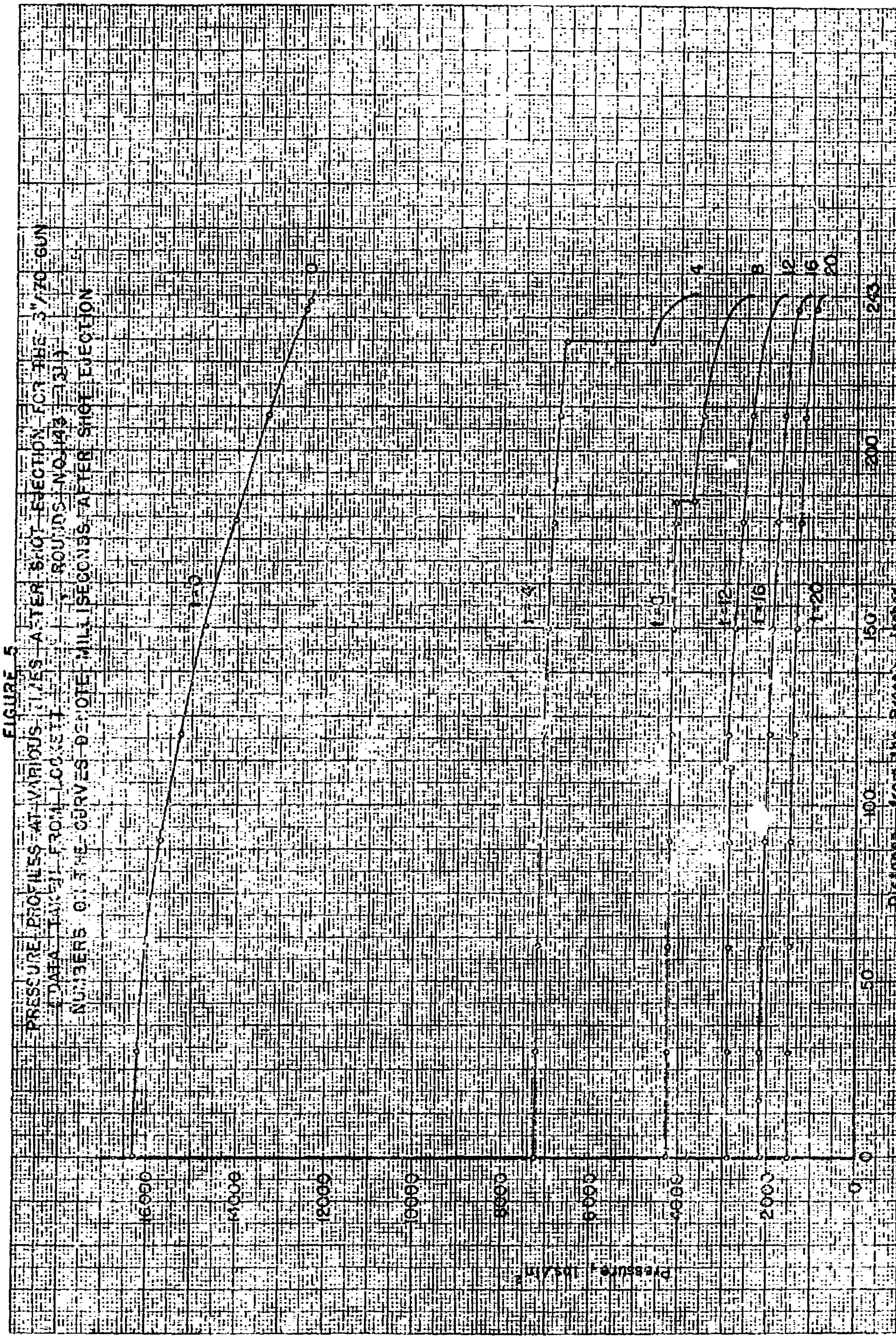
FIGURE 5

PRESSURE PROFILES AT VARIOUS TIMES AFTER SHOT EJECTION FOR THE 3"/70 GUN

(DATA TAKEN FROM LOCKETT ROUNDS NO. 143 - 131)

NUMBERS ON THE CURVES DENOTE MILLISECONDS AFTER SHOT EJECTION



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